

The axial form factor of the nucleon

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Abstract. The parity violation programs at MIT-Bates, Jefferson Lab and Mainz are presently focused on developing a better understanding of the sea-quark contributions to the vector matrix elements of nucleon structure. The success of these programs will allow precise semi-leptonic tests of the Standard Model such as that planned by the QWeak collaboration. In order to determine the vector matrix elements, a good understanding of the nucleon's axial vector form factor as seen by an electron, G_A^e , is also required. While the vector electroweak form factors provide information about the nucleon's charge and magnetism, the axial form factor is related to the nucleon's spin. Its $Q^2 = 0$ value at leading order, g_A , is well known from nucleon and nuclear beta decay, and its precise determination is of interest for tests of CKM unitarity. Most information about its Q^2 dependence comes from quasielastic neutrino scattering and from pion electroproduction, and a recent reanalysis of the neutrino data have brought these two types of measurements into excellent agreement. However, these experiments are not sensitive to additional higher order corrections, such as nucleon anapole contributions, that are present in parity-violating electron scattering. In this talk I will attempt to review what is presently known about the axial form factor and its various pieces including the higher order contributions, discuss the various experimental sectors, and give an update on its determination through PV electron scattering.

PACS. 12.15.Lk Neutral currents – 11.30.Er Charge conjugation, parity, time reversal, and other discrete symmetries – 13.60.-r Photon and charged-lepton interactions with hadrons – 13.15.+g Neutrino interactions – 14.20.Dh Protons and neutrons

1 Introduction

The neutral weak interaction between leptons and nucleons can be described by a set of three form factors that contain information about nucleon structure. The goal of present-day experiments in parity-violating electron scattering has been to determine the two vector weak form factors, G_E^Z and G_M^Z . These two form factors can be used along with the nucleon's electromagnetic form factors to disentangle the contributions of up, down and strange quarks to the nucleon's charge and magnetization distributions. However, in order to carry this out one also needs to know the third weak form factor coming from the nucleon's axial current, G_A^Z . The axial form factor has been determined at low momentum transfer in both quasielastic neutrino scattering and in pion electroproduction, but very little information on G_A^Z is available at momentum transfers greater than 1 (GeV/c)². In addition, the axial form factor as seen by an electron is substantially modified by electroweak radiative corrections that cannot yet be computed with high precision, including the term related to parity-violating coupling of a photon to the nucleon, known as the anapole coupling. Two experimental directions, quasielastic neutrino scattering and parity-violating quasielastic electron scattering from deuterium,

can improve our knowledge of both G_A^Z and of the anapole contribution.

2 Lepton–nucleon scattering

The nucleon electromagnetic current associated with lepton-nucleon scattering can be written as

$$\begin{aligned} \langle N' | J_\mu^\gamma | N \rangle = & \bar{u}_N \left[F_1^\gamma(q^2) \gamma_\mu + \frac{i \sigma_{\mu\nu} q^\nu}{2M_N} F_2^\gamma(q^2) \right. \\ & + \frac{G_F}{M_N^2} F_A^\gamma(q^2) (q^2 \gamma_\mu - q^\nu \gamma_\nu q_\mu) \gamma_5 \\ & \left. - \frac{i \sigma_{\mu\nu} q^\nu \gamma_5}{2M_N} F_E(q^2) \right] u_N, \end{aligned}$$

where F_1^γ and F_2^γ are the well-known Pauli and Dirac electromagnetic form factors, $q^2 = -Q^2$ is the four-momentum transferred from the lepton to the nucleon. The term containing the anapole form factor $F_A^\gamma(q^2)$ violates parity, and $F_E^\gamma(q^2)$ is a form factor that would arise with time-reversal violation. The anapole form factor has been computed by several authors [1, 2, 3] and is expected to be small at $Q^2 = 0$, but its computation is complicated by strong interaction effects in the nucleon, and its momentum transfer dependence is unknown. It is negligible in

electron scattering cross section measurements but it enters the asymmetry in parity-violating electron scattering at the same order as the weak nucleon axial form factor. The nucleon's neutral weak current is

$$\langle N' | J_\mu^Z + J_{\mu 5}^Z | N \rangle = \bar{u}_N \left[F_1^Z(q^2) \gamma_\mu + \frac{i \sigma_{\mu\nu} q^\nu}{2M_N} F_2^Z(q^2) + \gamma_\mu \gamma_5 G_A^Z(q^2) \right] u_N.$$

The vector form factors F_1^Z and F_2^Z are of primary interest in determining s -quark effects in PV electron scattering, and the axial form factor G_A^Z contains information about the nucleon spin. At leading order, G_A^Z can further be explicitly deconstructed using SU(3) symmetry into isovector and isospin singlet components to separate out the contribution of s -quarks to nucleon spin

$$G_A^Z(Q^2) = -\tau_3 G_A(Q^2) + G_A^s(Q^2),$$

where $\tau_3 = +1(-1)$ for $p(n)$, $G_A(0) = -(g_A/g_V) = 1.2670 \pm 0.0035$ [4] as determined in nucleon β decay, and $G_A^s(0) = \Delta s$, the strange quark spin content of the nucleon. The Q^2 dependence of G_A has generally been characterized by a dipole form, $1/(1 + Q^2/M_A^2)^2$, which can then be linked to a determination of an axial radius in a low momentum expansion of G_A with Q^2 :

$$\langle r_A^2 \rangle = -\frac{6}{g_a} \frac{dG_A}{dQ^2} \Big|_{Q^2=0} = \frac{12}{M_A}.$$

3 Available data

Two methods have been used to determine this lowest order Q^2 behavior of the axial form factor. The most direct method is to use quasielastic neutrino-nucleon scattering. Very little neutral current scattering data is available, so cross section data from the charged current process $\nu_\mu + n \rightarrow \mu^- + p$ has typically been used to extract M_A . Recently, a new global fit to neutrino data was carried out by Budd, *et al.* [5], which improved over earlier fits both by using the most recent determination of (g_A/g_V) along with new results for nucleon electromagnetic form factors. The improved fit gives $M_A = 1.001 \pm 0.020$ GeV. In pion electroproduction, M_A can also be extracted from the transverse component of the near-threshold electroproduction cross section by associating it with the electric dipole transition amplitude $E_{0+}^{(-)}$ through the low energy theorem of Nambu, Lurie and Shrauner [6] under the assumption that $m_\pi = 0$. A measurement was recently carried out at the Mainz Microtron [7], resulting in $M_A = 1.068 \pm 0.015$ GeV. In a recent topical review, Bernard *et al.* [8] used chiral perturbation theory to compute a finite mass correction to this extraction, which is substantial and results in a corrected M_A of 1.013 ± 0.015 GeV, bringing it into agreement with the neutrino data. Therefore, it appears that M_A is reasonably well determined and the low Q^2 behavior of G_A can at least be described phenomenologically. This does not, however, give a first principles theoretical description of $G_A(Q^2)$, nor does it provide an adequate

description at momentum transfers above 1 (GeV/c)². In addition, better modeling of neutrino scattering, guided by improved data, will be required for upcoming neutrino oscillation experiments. A new experiment, Minerva [9], has been proposed that would consist of a high granularity neutrino detector located at the NUMI beam line at Fermilab. This experiment would be able to provide a precise determination of G_A , including possible departures from the nominal dipole behavior, at $Q^2 < 2$ (GeV/c)² and a first determination of G_A for $Q^2 > 2$ (GeV/c)². Planning for another potential experiment is underway at Jefferson Lab, using the reaction $e + p \rightarrow \nu + n$, covering the range $Q^2 \sim 1 - 3$ (GeV/c)² [10]. This very challenging experiment would require detection of the recoiling neutrons at very forward angles, and the parity-violating asymmetry in the detected neutrons would be measured in order to constrain backgrounds.

While the above measurements would be able to better constrain $G_A(Q^2)$ and provide improved models of cross section data for neutrino oscillation experiments, it is the axial form factor as seen by an electron that is relevant to the parity violation program that is the topic of this workshop. The axial form factor seen in PV electron scattering can be written, going beyond first order, as

$$G_A^e(Q^2) = -\tau_3(1 + R_A^{T=1})G_A(Q^2) + R_A^{T=0}G_A^s(Q^2) + G_A^s(Q^2)$$

where $R_A^{T=0,1}$ are electroweak radiative corrections arising from higher order diagrams¹. The SU(3) octet form factor G_A^s is not present at tree-level, but appears once radiative corrections are included. Its $Q^2 = 0$ value can be estimated from the ratio of axial vector to vector couplings in hyperon β decay which, assuming SU(3) flavor symmetry, can be related to the octet axial charge a_8 and to the hyperon F and D coefficients [4],

$$G_A^s(0) = \frac{(3F - D)}{2\sqrt{3}} = \frac{1}{2}a_8 = 0.217 \pm 0.043.$$

Its Q^2 behavior has also not been measured, but it is usually assumed to have the same dipole form as the isovector form factor $G_A(Q^2)$ with the same mass parameter M_A .

It should be noted that while a decade of measurements related to the ‘‘spin crisis’’ have indirectly determined $G_A^s(0) = \Delta s$ from polarized deep-inelastic scattering, its Q^2 behavior is also unknown. There has been one determination of $G_A^s(0)$ from quasielastic neutrino scattering [12], which is in reasonable agreement with the polarized DIS data. An improved analysis of these data was carried out by Garvey *et al.* [13] who included possible effects of nonzero strange vector form factors. A recent further improved analysis was carried out by S. Pate [14], who combined the neutrino data with results from HAPPEX to perform a global fit to the three strange form factors (so far with only two constraints) to extract G_A^s at the mean of the two experiments, $Q^2 = 0.5$ (GeV/c)², rather than extrapolating to $Q^2 = 0$. A new direct measurement of G_A^s at low momentum transfer has been proposed using

¹ I am here following the notation in [11].

Table 1. Electroweak radiative corrections, computed in the \overline{MS} scheme, for the axial form factor measured in PV electron scattering. The values are taken from [1]

Source	$R_A^{T=1}$	$R_A^{T=0}$
1-quark	-0.18	0.07
anapole	-0.06 ± 0.24	0.01 ± 0.14
total	-0.24 ± 0.24	0.08 ± 0.14

the ratio of neutral current to charged current neutrino scattering at low momentum transfer, FiNeSSE, using a highly segmented detector with wavelength shifting optical fibers embedded in mineral oil to identify tracks left by the recoiling protons [15]. This experiment would potentially improve the determination of Δs by about a factor of two over the DIS data, and with less theoretical uncertainty.

Of more direct interest to the parity violation program is the radiative correction to the isovector $G_A(Q^2)$, of which the nucleon's anapole form factor F_A^γ is one component. The dominant contributions to $R_A^{T=0}$ and $R_A^{T=1}$ come from 1-quark terms such as γ - Z mixing and vertex corrections, which have been computed by several authors [1, 4]. Multi-quark or anapole contributions were also computed [1], who modeled them in terms of hadronic parity-violating NN couplings cast within a heavy baryon chiral perturbation theory framework. The results are shown in Table 1. While the 1-quark contributions dominate the correction, the anapole contributions dominate the uncertainty.

The axial form factor G_A^e , or at least its isovector piece $G_A^{e(T=1)}$, can be determined from the PV asymmetry in quasielastic scattering from deuterium, where the strange quark effects in the neutron and proton tend to cancel. Nuclear effects, including both parity conserving [17] and parity-violating [18, 19] contributions, have been shown to be small. The first measurement of $G_A^{e(T=1)}$ was carried out by the SAMPLE collaboration. The measured asymmetry at two momentum transfers are shown in Fig. 1. They agree fairly well with the calculation, which was carried out at $Q^2 = 0$, indicating that there is no anomalously large Q^2 dependence to the anapole term or to the correction at these very low momentum transfers. However, very little else is known about its behavior away from $Q^2 = 0$. Two model calculations [3, 2] of $F_A^\gamma(Q^2)$ have been carried out, and they indicate a much softer behavior with Q^2 than that of $G_A^Z(Q^2)$, even possibly an increase with Q^2 , as well as quite different behavior for the isoscalar and isovector pieces. These could substantially enhance the effects of radiative corrections at momentum transfers in the range of the G0 experiment. It would thus be very useful to have some experimental information on $G_A^{e(T=1)}$ at higher momentum transfers. A program of backward angle measurements with a deuterium target is part of the planned running for the G0 experiment, and aerogel Čerenkov detectors have been added to the detector array

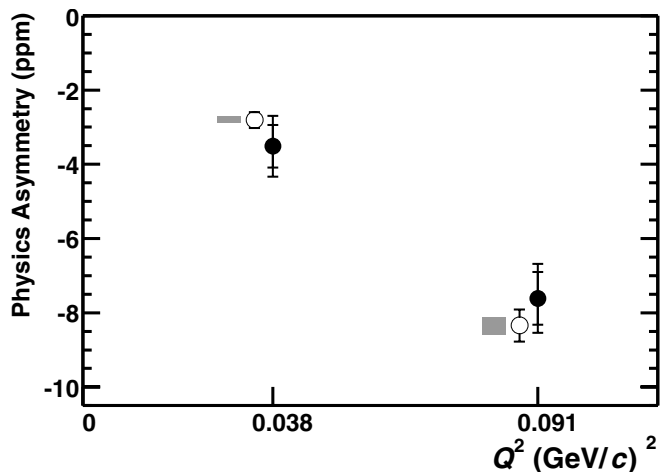


Fig. 1. Asymmetry results from the two SAMPLE deuterium experiments (*solid circles*, see [16]), compared to expectation from theory using the axial radiative corrections of [1] (*open circles*). The theory also assumes a value of G_M^s of 0.15 nuclear magnetons, and the *grey band* represents a change in G_M^s of ± 0.6 n.m.

in order to identify and separate charged pions produced in the deuterium target from the desired quasielastically scattered electrons. These data will not only reduce the model uncertainties in the determination of G_E^s and G_M^s from the hydrogen data, but will also allow the first experimental information on G_A^e away from the static limit. Shown in Fig. 2 are the projected uncertainties from the G0 experiment [20] in the difference between $G_A^{e(T=1)}$ and the tree-level G_A^Z that is seen in neutrino scattering, along with the calculation of [1] and the two SAMPLE measurements.

As an aside, it should be noted that, assuming that a determination of the nucleon axial form factor can straightforwardly be related to electron-quark interactions, the two SAMPLE measurements can be recast in terms of the two electron-quark couplings C_{2u} and C_{2d} . Prior to the SAMPLE measurements, experimental limits on these were from the original SLAC DIS parity-violation experiment [21], and from the parity-violating quasielastic electron scattering experiment on ^9Be carried out at Mainz [22]. The two SAMPLE measurements are sensitive to the combination $C_{2u} - C_{2d}$. These are modified by 1-quark radiative corrections, and in the case of elastic e - N scattering the multi-quark corrections as well. In order to compare directly to the SLAC DIS data, the multi-quark radiative corrections must be removed, which although small, dominate the uncertainty. The resulting values from the 200 MeV and 125 MeV data sets, respectively, are

$$C_{2u} - C_{2d} = -0.042 \pm 0.040 \pm 0.035 \pm 0.02$$

$$C_{2u} - C_{2d} = -0.12 \pm 0.05 \pm 0.05 \pm 0.02 \pm 0.01,$$

where the first two uncertainties are statistical and experimental systematic, the third is that due the radiative corrections, and, for the 125 MeV case, the last corresponds to variations in G_M^s by ± 0.6 because it is unde-

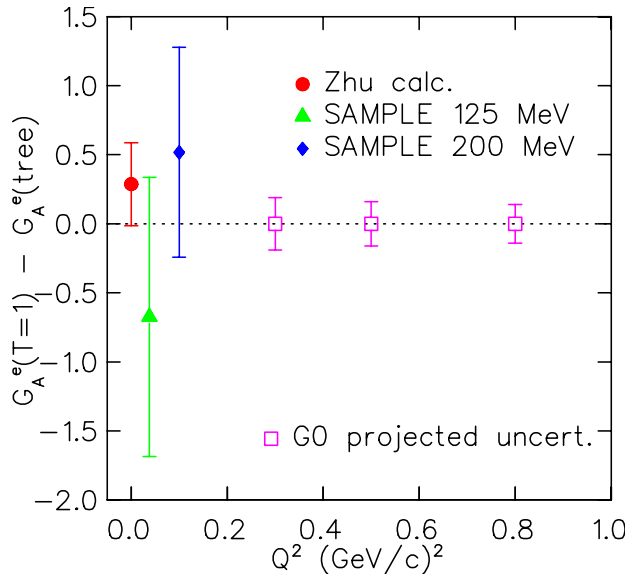


Fig. 2. The difference between the axial form factor seen by an electron in parity-violating electron scattering and $G_A^Z(Q^2)$, the tree-level form factor. Shown are the calculation of [1], SAMPLE data, and projected uncertainties in the G0 experiment

terminated at this momentum transfer. These values are in good agreement with the Standard Model prediction [4], and represent a significant improvement over the earlier data.

4 Conclusion

In summary, while much attention has been focused on determination of the neutral weak vector form factors, in order to extract strange quark effects in the nucleon, there are variety of experimental avenues to pursue in the near future to improve our knowledge of the nucleon's axial form factor. The tree-level form factor is now known reasonably well at low momentum transfers from neutron beta decay, from quasielastic neutrino scattering and from pion electroproduction. Its knowledge at higher momentum transfer, including potential deviations from a generic dipole behavior, can be improved with new neutrino scattering experiments, and these experiments will help provide the required precision cross section information needed for the next generation of neutrino oscillation

measurements. In PV electron scattering, the axial form factor is substantially modified and very little is known about the Q^2 behavior of the higher order terms. The G0 experiment will uniquely be able to provide the higher Q^2 data through quasielastic scattering from a deuterium target.

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